# Fermi Calculations 

Dr. Douglas G. Frank

Enrico Fermi was a nuclear physicist famous during the first half of the $20^{\text {th }}$ century. He was a key player in the Manhattan Project, and won the Physics Nobel Prize in $1938{ }^{1}$.

He was particularly good at making approximations, a very valuable skill in science since you seldom have all of the data you need in a given investigation. He was so good at it that he appeared on a radio quiz program where people would call in and ask him questions like, "How many piano tuners are there in New York City?" People lost interest in the program
 because he always won...his guesses were always within $10 \%$.

Fermi is also remembered for the 'Fermi Paradox,' where Fermi calculated the probability of there being extra-terrestrial life ${ }^{2}$. If the probability is so high, he wondered, where is everyone?

Fermi Calculations are not only fun, they are very practical for guiding your thoughts on policy and informing your decision making. For example, does it change your thinking about world population to know that everyone in the world could live in a suburban home in Texas (two per house) and be fed by the agricultural output of Iowa? Does it guide your career choices if you know that the minimum wage is about $\$ 8$ and there are about 2000 hours in the work year, so a young couple who both work full time are going to be making (before taxes) around $\$ 32 \mathrm{k}$ ?

In situations where thinking on your feet is a definite advantage (such as negotiations and interviews), being able to make fast and accurate predictions is a valuable skill.

In performing Fermi Calculations ("calculated approximations") ${ }^{3}$, there are some basic principles and techniques that will improve your accuracy:
a) Start by making a guess and writing it down, preferably on the back of an envelope or napkin. (You will often hear scientists and engineers refer to 'back of the envelope calculations,' or "We designed it on a napkin.")
b) Then, refine your guess by identifying the components in your guess, making educated approximations of those components, and then recombining the approximations to obtain a new result. The more approximations you combine, the closer your final approximation is likely to be to the actual value. This is counterintuitive at first (If I guess more I get more accurate?), but multiple guesses will have the tendency to converge; i.e., high guesses will compensate for low ones.
c) If you are having difficulty coming up with approximations, try breaking them down into smaller chunks; i.e. perform Fermi Calculations to obtain your approximations!
d) If you have very little idea how to guess on a certain value, try guessing the upper and lower bounds of the value, and use the geometric mean of those bounds. (The geometric mean of two numbers is the square root of their product.)

[^0]e) If your having trouble guessing bounds, try thinking of similar items to guide your guesses. For example, you know that there are 52 cards in a deck and that a deck of cards stands less than an inch high, so a stack of 52 papers is also likely to be less than an inch high.
f) Round your guesses to one significant figure. In other words, make the math easy! Rounding to the nearest ten and dividing by two is fast. For example, you know that the U.S. is shaped like a rectangle and that there are about 50 states, so the U.S. is about ten states wide by five states high.
g) Once you've obtained what you think is a pretty good guess, sanity check it by comparing it to some related value. For example, check a map, the yellow pages, or look it up on Google. Remember, many reported values are just educated guesses! If you are within a factor of two, you're doing well!

## Practice Problems

- How much television do you watch in a year? How much time in front of a computer?
- How many dogs live in your city?
- How many times will your heart beat in your lifetime?
- How many people are you likely to date before you get married?
- How many children are you likely to have?
- How much money are you likely to make in your lifetime?


[^0]:    ${ }^{1}$ Wikipedia, "Enrico Fermi"
    ${ }^{2}$ S. Webb, If the Universe Is Teeming with Aliens... Where Is Everybody? Fifty Solutions to Fermi's Paradox and the Problem of Extraterrestrial Life, Springer, 2002.
    ${ }^{3}$ L. Weinstein and J.A. Adam, Guesstimation, Princeton University Press, 2008.

